Application Of The Method Of Finite Elements For Investigation Of The Dynamic Stress-Deformed Condition Of Pipeline Sides When Exposed To External Loads

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Abstract.
In work with the help of the finite element method (FEM), a numerical solution of stresses and displacements in the form of graphs taking into account different boundary conditions and geometric parameters was obtained. The conducted study showed a significant effect of the accounted effects and boundary conditions on the values of dynamic characteristics and the static stability of pipelines.
***Keywords**: pipeline, static stability, finite elements, stresses, displacements.*

Introduction

Trunk pipelines and pipelines of enterprises in the energy, petrochemical and other industries constitute a fairly large part of their tangible assets. As a rule, pipelines are very highly loaded structures, because even in their design, in order to save metal, the lowest safety factors are actually laid. This requires a very precise justification of the strength and resource for all possible types of loading. Carrying out such an analysis is impossible without the use of modern computer systems. At the same time, the calculator needs to understand the nature of the solution beforehand, and the numerical results should only clarify some of the coefficients. It is important to know the features of the deformation of the structure with its geometrically nonlinear behavior, considering that even for ideally elastic material a slight increase in the load can lead to uncontrolled growth of strains and stresses [1,2,3,4]. Without understanding the specific features of the deformation of various elements achieved through analytical modeling, it is impossible to use the provisions of these standards. To solve the above problem, the finite element method was used [5,6] and computer simulation.

Problem formulation methods of solution. The mathematical model of the system is based on the dynamic three-dimensional equations of the linear theory of viscoelasticity according to the rheological Kelvin-Voigt model [6]:

$$\nabla\cdot\boldsymbol{\sigma}+\mathbf{f}_V=\rho\frac{\partial^2\mathbf{u}}{\partial t^2},\ \boldsymbol{\varepsilon}=(\nabla\mathbf{u})^S,\ \boldsymbol{\sigma}=\lambda\theta\mathbf{E}+2\mu\ \varepsilon+2\eta\dot{\mathbf{e}},$$

where $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$ – tensors of stresses and deformations; $\mathbf{e}$ – strain tensor deviator $\mathbf{e}=\boldsymbol{\varepsilon}-\theta/3\,\mathbf{E}$; $\mathbf{E}$ – unit tensor; $\mathbf{u}=\mathbf{u}(\mathbf{r},t)$, $\mathbf{f}_V=\mathbf{f}_V(\mathbf{r},t)$ – the vectors of displacements and given volume forces, which depend on time t and the radius of the point vector of the continuous medium $\mathbf{r}$; λ, μ, η – Lame coefficients and material viscosity; ρ – density; θ – volumetric deformation; ∇ – differential operator of Hamilton. The above differential formulation of the problem of the theory of viscoelasticity is equivalent to a variational formulation in the form of the principle of possible displacements:

$$\int_V\delta\mathbf{u}\cdot\mathbf{f}_v\,dV+\int_S\delta\mathbf{u}\cdot\mathbf{f}_S\,dS+\int_V\delta\mathbf{u}\cdot\mathbf{f}_{inert}\,dV=\delta\pi,$$

where $\delta\mathbf{u}$ – vector of possible displacements of points in a continuous medium; The integrals on the left represent the work of external volume and surface forces,

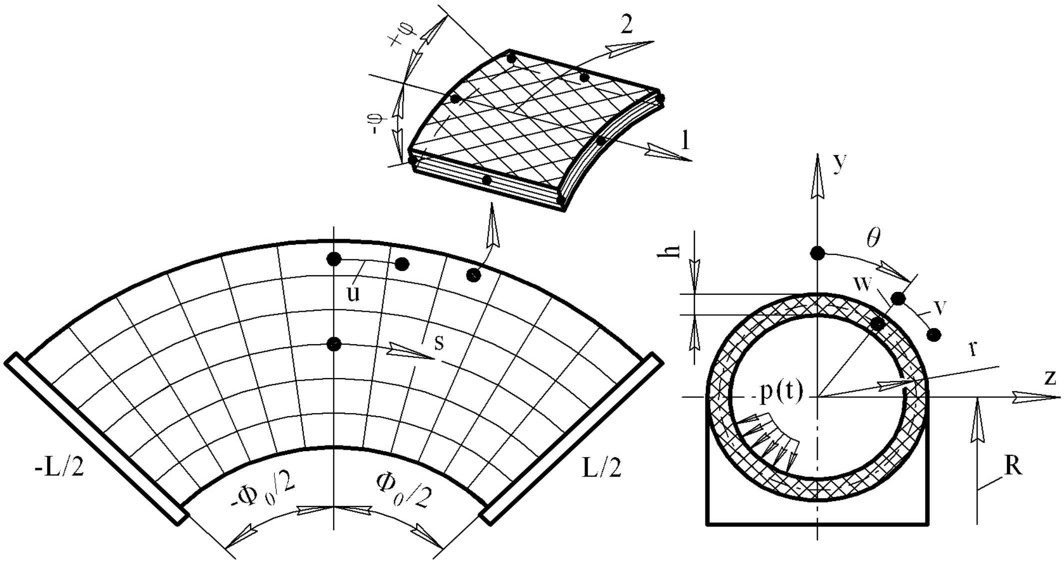


Fig.1. Calculation scheme

as well as inertia forces $\mathbf{f}_{inert} = -\rho \dfrac{\partial^2 \mathbf{u}}{\partial t^2}$ on possible displacements; $\delta\pi$ – virtual work of the internal forces in the deformation of the body: $\delta\pi = \int_V \delta\boldsymbol{\varepsilon} \cdot\cdot \boldsymbol{\sigma}\, dV$. For the numerical solution of this equation, the standard procedure of finite-element discretization with respect to spatial variables using the node isoparametric element was applied [7]. Taking into account the form of the defining relation recorded in the matrix form using matrices of elastic and viscous modules **D** and **S**:

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon} + \mathbf{S}\dot{\boldsymbol{\varepsilon}},$$

The following matrix time differential equation is obtained:

$$\mathbf{M}\frac{d^2\mathbf{U}}{dt^2} + \mathbf{R}\frac{d\mathbf{U}}{dt} + \mathbf{K}\mathbf{U} = \mathbf{F}(t),$$

where $\mathbf{U} = \mathbf{U}(t)$, $\mathbf{F} = \mathbf{F}(t)$ – global vectors of nodal displacements and nodal forces as functions of time (or temperature loads), **K, M, R** are global stiffness, mass, and dissipation matrices. The FEM procedures provide for a transition from differential dependencies, for individual finite elements to a global system of equations for the entire array.

Further transformation is based on the standard approach used in the linear theory of elastic oscillations [7]. Suppose that the external load varies in time according to the harmonic law, which can be written in the form:

$$\mathbf{F}(t) = \mathbf{F}e^{i\omega t},\ \mathbf{F} \in R,$$

where **F**, ω – amplitude and circular frequency of the force, i is the imaginary unit.

Then a particular solution, corresponding to the steady-state forced oscillations of the system, is sought in an analogous form:

$$\mathbf{U}(t) = \mathbf{U}e^{i\omega t},\ \mathbf{U} = \mathbf{U}_1 + i\mathbf{U}_2,$$

where U is the global vector of complex amplitudes of nodal displacements; $\mathbf{U}_1$ and $\mathbf{U}_2$ – Respectively, the real and imaginary parts. Substituting expressions of force and displacement in the differential equation, we obtain a system of complex algebraic equations with respect to the vector of complex amplitudes of nodal displacements:

$$(-\mathbf{M}\omega^2 + \mathbf{R}i\omega + \mathbf{K})\mathbf{U} = \mathbf{F}.$$

After solving the last system, the amplitude values of the node accelerations are calculated according to the following formula:

$$w = \omega^2 \sqrt{\mathbf{U}_1^2 + \mathbf{U}_2^2}.$$

To begin with, the equation of dynamics for curvilinear rods is derived. A feature of the design work under dynamic action is the need to take into account the inertia forces associated with the relative displacement of the point of the deformed body. The presented algorithm is implemented in the software complex MAPLE and allows to calculate viscoelastic steady-state oscillations of mechanical structures under the influence of external harmonic force. It is known that if the stress concentration is not considered, then a small number of finite elements is sufficient to describe elastic vibrations. Taking into account that in the model high-order elements with quadratic approximation of displacements are used, only 156 finite elements were used.

The material is characterized by a modulus of elasticity $E = 2{,}06 \cdot 10^5$ MPa, the shear modulus $G = 8{,}0 \cdot 10^4$ MPa and density $\rho = 7000$ kg/m^3. One of the sources of elastic oscillations of pipes and pipelines is the pulsation of the pressure of the working medium. Under the influence of pulsations, the tube performs ordinary forced oscillations associated with the expansion-contraction of the wall. Under certain conditions, stationary oscillation regimes become dynamically unstable, parametric resonance develops in the system. The results of studies of the stability of the axisymmetric modes of oscillations of straight pipes, such as thin-walled cylindrical shells, are presented in numerous papers, for example, [10]. A distinctive feature of the parametric oscillations of curvilinear pipes is the presence along with simple resonances of combination resonances of the total type [10]. Consider the pipe (Fig. 1), the axial line of which represents an arc of a circle of radius R, long L, with a central angle (bending angle) Φ_0. The pipe has an ideally circular cross-section of radius r, moreover $r/R \le 1/5$. Wall thickness h. The pipe is under the influence of monoharmonic pressure $p(t) = p_m(1+\psi\cos\Omega t)$, where p_m – Mean (working) pressure, $\psi = p_0/p_m$ – Ripple parameter; p_0 and Ω – respectively, amplitude and circular frequency. The perturbed form corresponding to the deviation of the motion from the unperturbed one is approximated by functions of the form:

$$w(s,\theta,t) = \sum_{n=1}^{\infty}\left[\tilde{w}_{1n}(s,t)\cos n\theta + \tilde{w}_{2n}(s,t)\sin n\theta\right],$$

$$v(s,\theta,t) = -\sum_{n=1}^{\infty}\frac{1}{n}\left[\tilde{w}_{1n}(s,t)\sin n\theta - \tilde{w}_{2n}(s,t)\cos n\theta\right], \qquad (1)$$

$$\tilde{w}_{m1}(s,t) = w_{m1}\,(1-\cos\frac{\pi s}{L}), \quad \tilde{w}_{mn}(s,t) = w_{mn}\cos\frac{\pi s}{L}.$$

Here v and w – moving points of the middle surface in the circumferential and radial directions, s and θ – axial and circumferential coordinates, t – time, $w_{mn} = w_{mn}(t)$ – generalized coordinates corresponding to the rod (m = 1, 2 and n = 1) and shell (m = 1, 2 and n = 2, 3, 4, …) forms. Index m = 1 corresponds to oscillations in the plane of the tube, the index m = 2 – the oscillations along the normal to the plane. The rod (beam) shape reflects the movements associated with the movements of the cross-section of the pipe as a rigid whole, the shell forms are movements associated with the deformation of the shell wall. The n = 2, 3, 4, .., ∞ waves in the circumferential direction and one half-wave in the axial direction.

On the basis of the approximation (1), the semi-instantaneous theory of anisotropic layered shells and the Lagrange equations of the second kind, two independent systems of coupled differential equations with variable stiffness coefficients

$$[A]\{\ddot{w}\}+2\varepsilon[B]\{\dot{w}\}+\alpha([C]-2\mu[F]\cos\Omega t)\{w\}=0, \qquad (2)$$

Which describe the parametric oscillations of the tube, both in the plane of its curvature, and along the normal to the plane. In this case, $\alpha=6\pi D_2/(m_T r^3)$ – factor, $m_T=2\pi\rho hr$ – weight of pipe length unit, $D_2=E_2h^3/\left(12\left(1-\nu_{12}\nu_{21}\right)\right)$ – bending rigidity of the wall in the circumferential direction, E_2, ν_{12}, ν_{21} – effective elastic constants [11], $\mu=0.5p_0/p_{кр}$ – coefficient of parametric excitation, $p_{кр}=3D_2/r^3$ – critical external pressure corresponding to the static loss of stability, - damping factor, $f_{nn}=n^2-1$. Matrix elements $[A]$, $[B]$, $[C]$ are determined by the recurrence formulas [12].

The resolving system of equations (2) describes the parametric oscillations of the bound shell-rod system. The source of parametric excitation is a periodic change in the volume of the internal cavity. At the same time, the pressure "works" not on the basic (axisymmetric) displacements, but on additional displacements associated with bending deformations of the wall.

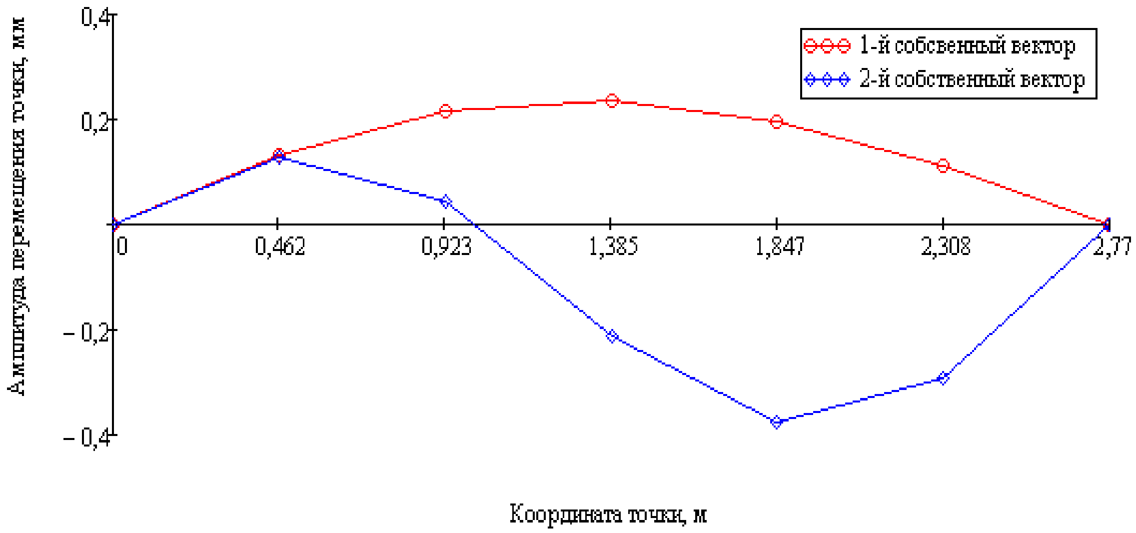


Fig. 2. Graph of changes in eigenvectors.

From the analysis of the structure of the matrix $[C]$ it follows that the rod form (n = 1) is associated with the shell form (n = 2). This means that the vibrations of the tube as a rod are accompanied by oscillations of the shell wall, associated with flattening of the cross section (the Karman effect is manifested). The interaction of the core and shell forms is due to elastic bonds, the intensity of which is characterized by off-diagonal elements of the matrix $[C]$ and depends on the length of the pipe L and the curvature parameter r/R. The shorter the tube and the larger the parameter r/R, the stronger these links are. In addition, the shell forms interact with each other: separately n – even harmonics (n = 2, 4, 6, …) and n – Odd harmonics (n = 3, 5, 7, …).

Figure 2 shows the 1st and 2nd forms of oscillations of a rectilinear fixed rod ($\mathbf{F}=0$). It can be seen that the results of the calculations are consistent with the data [9].

Example 1. The section of the pipeline AB with some initial curvature is fixed at the ends A and B and is under its own weight. Determine the stress-strain state at various pressures and temperatures. The pipeline is under the influence of its own weight q, including the weight of the product, as well as the internal pressure P. The temperature regime is determined by the initial temperature T_0, at which the pipeline was fixed in this state, and the operating temperature $T_Э$.

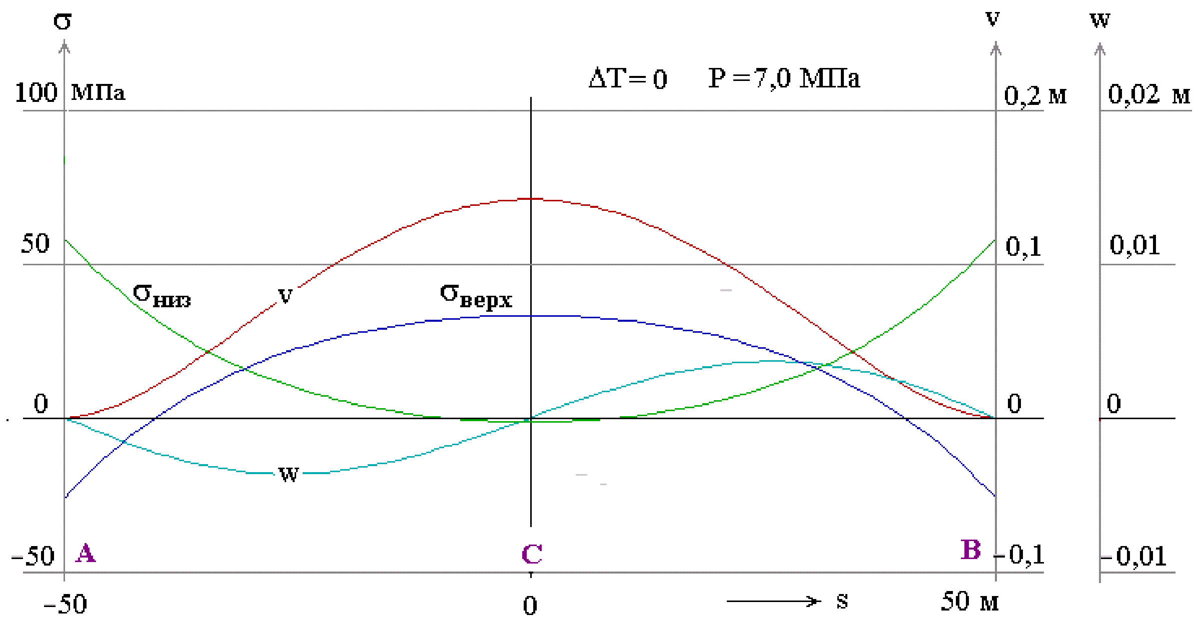


Figure 3. Distribution of displacements of v, w and longitudinal stresses along the upper and lower generatrix of the pipeline section with constant initial curvature ($\rho = 300\,m$) under the influence of its own weight.

Analyzing these results, it can be seen that, in contrast to straight beams in the presence of curvature, the bending moment can change sign (bending direction) depending on the ratio of temperature and pressure. The curves reduce the stresses due to the appearance of freedom of longitudinal displacements. In this phenomenon, in particular, the work of compensators is based.

The smaller the radius of curvature R and more number n, the stronger the interaction. In the conditions of the nominal operating mode, we regard an ideal pipe as a parametrically excited system with a small parameter depth μ. Analysis of the stability of elastic vibrations is limited to the region of lower eigenfrequencies. To calculate the boundaries of instability regions for the main simple resonances $\Omega = 2\omega_i$ and the main combination resonances $\Omega = \omega_i + \omega_j$ (i, j = 1, 2, .., 5) we use the small parameter method [4]. The perturbed form of motion is represented as a superposition of proper forms. To solve the eigenvalue problem, we use the Jacobi method. The report presents the results of a study of the dependence of the spectra of the lowest eigenmodes and frequencies on the working pressure, as well as geometric, structural and technological factors. The obtained results are compared with the data of the FEM calculations.

The pictures of resonance bands are shown depending on the angle Φ_0 = 5º, 90º, 135º, 180º (at L = const) and reinforcement angles φ = ±55º, ±65º, ±75º. It is established that when the curvature of the tube decreases, the lowest natural frequency ω_1 increases, and higher frequencies ω_2, ω_3, ω_4, ω_5 on the contrary, decrease and approach the natural frequencies of the straight tube. In this case, the regions of dynamic instability shift towards smaller values $\Omega/(2\omega_1)$, the relative width of the bands narrows. At Φ_0 = 5º Region of instability $2\omega_1$ and $\omega_1 + \omega_2$ practically disappear.

With increasing reinforcement angles $\pm\varphi$ The regions of dynamic instability corresponding to the main simple and principal Raman resonances are shifted toward higher values $\Omega/(2\omega_1)$. In this case, the regions of instability $2\omega_1$ and $\omega_1 + \omega_2$ narrowed, and the regions of instability $2\omega_2$, $2\omega_3$, $2\omega_4$, $2\omega_5$ and $\omega_2 + \omega_4$, $\omega_3 + \omega_5$, on the contrary, expand.

Таблица 1

The values of the critical bending moment M_{cr} for a tube with initial curvature of the axis

λ_0	M_{cr}		
	The results obtained	**Literary source**	
		[7]	[8]
0	0,322	-	0,321
0,1	0,282	0,281	0,285
0,2	0,247	0,254	0,254
0,3	0,167	0,191	0,190

In Table. 1 represents the values of the critical bending moment calculated on the basis of the refined solution M_{cr} for pipes with initial curvature of the axis R_0. As might be expected, with an increase in the initial curvature, the values of the critical moment decrease. Let us construct the dependence of the critical bending moment on the pressure in the normalized coordinates ***m*** and ***q*** (Fig. 2). The graph represents the limiting curve **1**, characterizing the combination of loading, leading to a loss of stability. For points that lie in the plane bounded by this curve, the loading level by moment and pressure does not correspond to the loss of stability. The points above the curve correspond to a combination of loads in which the loss of stability of the pipe is unavoidable.

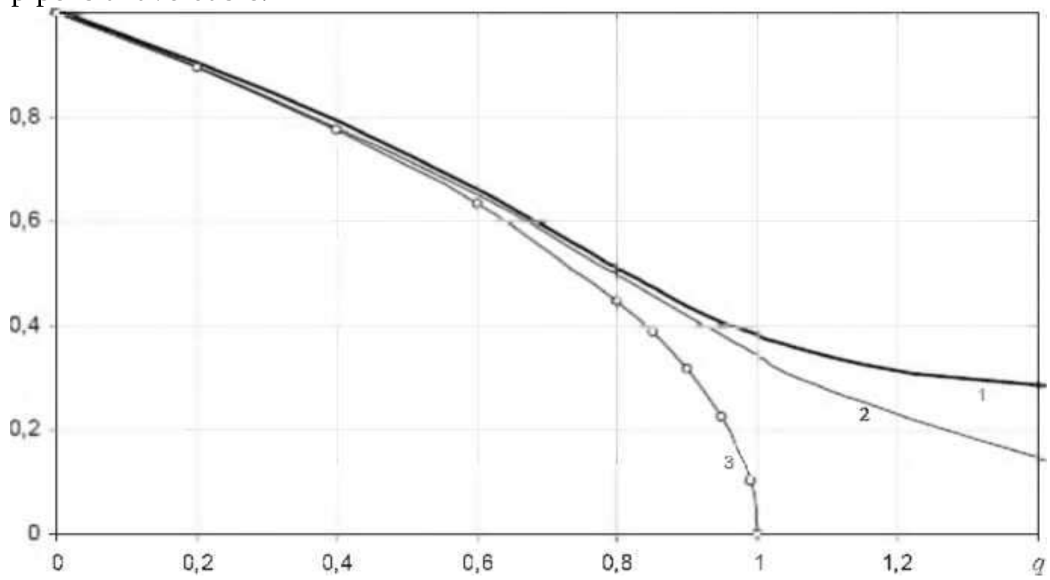
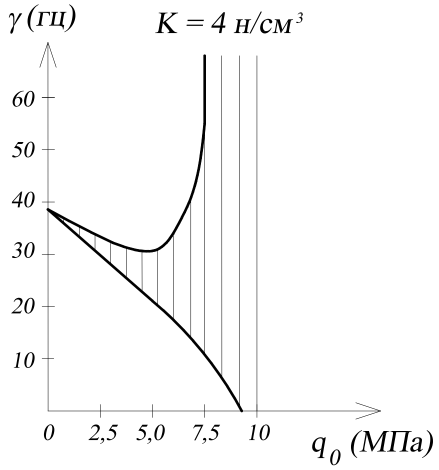


Fig. 3. Limit curves with the joint action of bending moment and pressure for a straight pipe: 1 - our results; 2 - the data [9]; 3 - data [8].

In the region q <0 (internal pressure), all the graphs practically coincide, so in Fig. 3 they are given for the region q> 0.

Figure 4.5 shows the results of calculations of the region of dynamic instability of the pipeline for a metal pipe 1420x10 mm, for a pulsating flow of liquid. The method for estimating the dynamic stability of a pipeline reduces to finding the position of the point (γ, q_0). If this point falls on a plane free from the shaded areas of instability, then the stability of this pipeline is ensured. The smallest frequency of proper bending oscillations $min\,\omega_{mn}$, for all the types of anchoring of the ends of pipeline sections at $m=2$ and $n=1$, is shown in Figure 6. those. $min\,\omega_{mn}=\omega_{21}$.



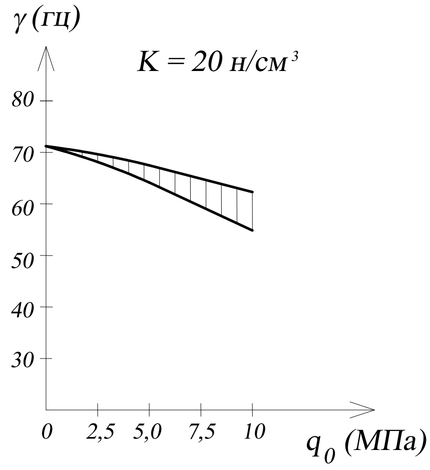


Fig. 4 and 5. The regions of dynamic instability of the deep-water gas pipeline for a metal pipe of 1420x10 mm and a protective reinforced concrete layer h_1=30 mm.

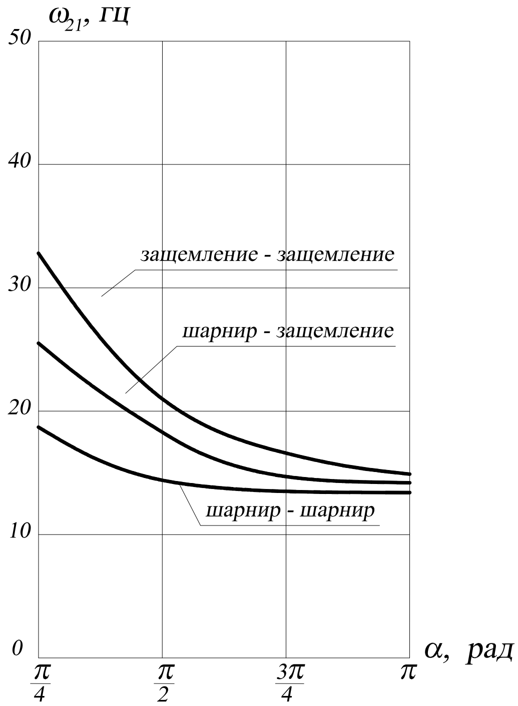


Fig.6 Dependence ω_{21} from the corner α at a relative thickness $h/r=1/70$ With different types of fixation of the end sections.

Conclusions

1. The smallest frequency of proper bending oscillations $min\,\omega_{mn}$ It is realized for all considered types of fastening of the ends of sections of pipelines on the second shell form of oscillations at $m=2$ and $n=1$, i.e. $min\,\omega_{mn}=\omega_{21}$.

2. With an increase in the pipe curvature parameter μ frequencies of proper flexural vibrations of pipeline sections ω_{mn} at $m=1, 2, 3$ and $n=1$ Essentially increase under any conditions of fastening of the ends.

3. The pressure prevents the deformation of the cross sections of the pipelines and thereby increases their rigidity, which leads to an increase in the frequencies at any fixation of the end sections of the sections. Most Frequency ω_{21} increase in thinner and flat tubes $(h/r=1/70$ and $\mu=6)$. With the growth of internal pressure p_0 from 0 to 2 MPa, the frequencies are almost doubled.

List of references

1. Auliyakulov N.N., Safarov I.I.. Modern problems of statics and dynamics of underground pipelines. Tashkent. 2007. 306 p.
2. Ainbinder A.B., Kamershtein A.G.. Calculation of trunk pipelines for strength and stability: reference manual. M .: Nedra, 1982. 341 p.
3. Aleshin V. V. [and others] Numerical analysis of the strength of underground pipelines. Moscow: Izd-vo Uedrsial URSS, 2003. 320 p.
4. Bazarov M.B. Safarov I.I., Shokin Yu.M. Numerical simulation of oscillations of dissipative-inhomogeneous and homogeneous mechanical systems.-Novosibirsk, Siberian Branch., 1996. -189 p.
5. Zenkevich O. Finite Element Method in Engineering. Moscow: Mir, 1975.-634p.
6. API 579. Recommended Practice Fitness-for-Service, First Edition. - American Petroleum Institute, January 2000.
7. Sokolov V.G.. Free oscillations of a pipeline with a fluid flow compressed by a longitudinal force, under various boundary conditions at the ends // Vestnik civil engineers. - 2010. - №2 (23). - P. 61-64.
8. Rashidov T.R., Safarov I.I. And others. Two main methods for studying the seismic stress of underground structures under the action of seismic waves. Tashkent: DAN. № 6, 1989. p. 13-17.
9. Efimov A.A. Dynamic stability of steel gas pipelines under underwater laying [Text] / Sokolov V.G., Efimov A.A. // Proceedings of universities. Oil and gas. №4, 2007, p. 47-51.
10. L.A. Rozin, Fundamentals of the finite element method in the theory of elasticity. L .: Izd-vo LPI, 1972. 77 p.
11. Safarov I.I., Teshaev M.H., Esanov N.N., Kamroeva Z.N. "Mathematical modeling of intrinsic and forced oscillations of curvilinear tubes interacting with the medium. Toshkent, 2009.-161 p.
12. Seleznev V. Ye., Aleshin V. V., Klishin G.S.. Methods and technologies of numerical simulation of gas-conducting systems. Moscow, 2002. 448 p.